MEASUREMENT OF AMPLIFICATION COEFFICIENTS

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This study will perform an analysis of errors occurring in the measurement of the coefficient of amplification of the optical medium of a laser by the method of irradiation with monochromatic laser radiation. These errors are related to lack of correspondence in parameters of the probe laser and the laser to be measured.

1. Formulation of the Problem

The coefficient of amplification is the basic parameter characterizing the active medium of a laser. In the great majority of cases the coefficient of amplification is measured by irradiation of the active medium with monochromatic radiation produced by the same quantum transition as that employed in the laser being studied. In many cases the radiation source is another laser using the same laser transition as the active medium being measured, but with other parameters (density, temperature, component concentration) possibly greatly different.

The difference in parameters between the probe and the medium measured inescapably leads to differences between the contours of the probe and studied lines. When using a laser as the probe, source-generation mode structure must also be considered.

The purpose of the present study is to perform an analysis of errors which may develop in measurement of the coefficient of amplification due to the above-mentioned variations and to formulate conditions for minimization of such errors. We will make two preliminary remarks concerning the ambiguity of the term "coefficient of amplification"* itself. Frequently, in the literature the concepts of coefficient of amplification K and unsaturated index of amplification $\varkappa$ are confused. While K is a dimensionless quantity, characterizing the effect over the entire active medium $K = P_-/P_+$, where P_- and P_+ are output and input power, $\varkappa$ characterizes the amplification per unit length, reduced to an infinitely thin layer, i.e., the local amplification. The two quantities are related as follows:

$$K = \exp\left[(\varkappa - \delta) L_0\right] \tag{1.1}$$

where δ is the index of distributed losses (scattering), and L_0 is the length of the active medium.

Another ambiguity is connected with interpretation of the coefficient of amplification without reference to the generation mode in which it is proposed to utilize the active medium studied. If the generation contour is significantly narrower than the amplification contour, it is natural to treat this term as being the amplification coefficient for monochromatic light. Such a situation occurs in single-mode (with respect to axial modes) operation and also, almost always, in solid-state lasers. On the other hand, if the generation contour is comparable with the amplification line contour (for example, in an argon laser without mode selection), the term should obviously correspond to some total amplification over the contour. For the sake of brevity, we will refer below simply to single and multimode regimes, considering gas lasers at pressures below atmospheric. Thus, the measured value of the coefficient of amplification is determined by the degree of overlapping of the probe and studied lines, and its true value depends on the generation regime (Fig. 1). In the general case four variants of the measurement process must be analyzed, depend-

* Here and below we consider the unsaturated coefficient of amplification.

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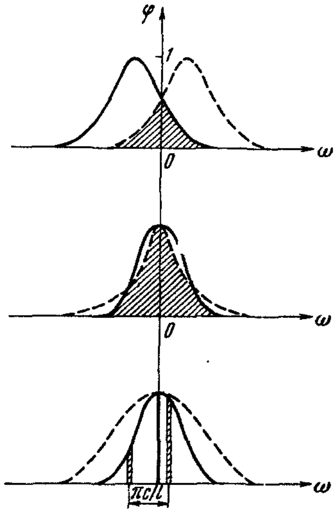

Fig. 1

ing on the mode structure of the probe laser and the laser using the material being studied.

2. Single-Mode Laser Irradiating Active Medium of Another Single-Mode Laser

Since deviations of the laser frequency from the center of the amplification line may vary over quite wide limits, we shall introduce certain assumptions for the sake of simplification. We will concern ourselves with the value of the amplification coefficient at the center of the amplification line of the active medium studied. This assumption is natural, since this value, generally speaking, corresponds to the maximum of the amplification coefficient. As for the generation frequency of the probe laser, here we must commence with the frequency interval between axial modes $\Delta\nu = c/2L$, where c is the speed of light and L, the length of the resonator. The goal of mode selection, as a rule, is the choice of the mode closest to the center of the line. With this assumption it is obvious that deviations of the generation frequency from the center of the amplification line must not exceed one half of the interval mentioned, namely, $\Delta\nu' = c/4L$. We will now introduce the function $\varphi(\omega)$ describing the contour of the amplification line and normalized such that at the line center $\varphi(0) = 1$. Inasmuch as $\varkappa(\omega) \sim \varphi(\omega)$ the maximum relative error in measurement of the index of amplification connected with uncertainty in the probe-laser frequency will be given by

$$\varepsilon = \frac{\varphi(0) - \varphi(\omega_M)}{\varphi(\omega_M)} = \frac{1}{\varphi(c/4L)} - 1 \tag{2.1}$$

where ω_M is the mode frequency. It is known [1] that for a homogeneously broadened contour

$$\varphi(\omega) = \frac{1}{1 + 4(\omega/\Delta\omega_e)^2} \tag{2.2}$$

and for a Doppler contour

$$\varphi(\omega) = \exp\left[-4\ln 2\left(\frac{\omega}{\Delta\omega_d}\right)^2\right] \tag{2.3}$$

where $\Delta\omega$ is the half-width of the contour. In as much as usually $L \approx 1$ m, we have $\varphi(c/4L) \approx \varphi(10^8\,\mathrm{Hz})$. Figure 2 depicts the dependence of relative error on amplification line half-width for homogeneous and Doppler contours. Independent of the character of the widening at $\omega_M = 0$ the error $\varepsilon = 0$, and if $\omega_M \gg \omega$, then $\varepsilon \to \infty$.

3. Multimode Laser Irradiating Active Medium of Another Multimode Laser.

In the presence of a large number of axial modes the index of amplification of the medium must have a value close to that which would prevail in the case of amplification over the entire contour $\varphi_2(\omega)$. Analogous concepts apply to the generation contour $\varphi_1(\omega)$ of the probe laser. The spectral index of amplification of the medium is [1]

$$\varkappa_2(\omega) = \frac{J_{ik}\varphi_2(\omega)}{\int\limits_\omega \varphi_2(\omega)\,d\omega}, \qquad J_{ik} = \frac{\pi c^2 A_{ik}}{2\omega_0^2}\left(N_i - \frac{g_i}{g_k}N_k\right) \tag{3.1}$$

where A_{ik} is the Einstein coefficient for emission, ω_0 is the line center frequency, N_i and N_k are the populations of the upper and lower levels, and g_i and g_k are the statistical weights of the same levels.

We will now define the value of the index of amplification averaged over the entire line which is actually observed in amplification measurements. It is obvious that this value cannot be determined without reference to the spectral composition of the probe radiation.

The averaged index of amplification, considering Eq. (3.1) and the degree of spectral overlap, may be written as

$$\varkappa_2 = J_{ik}\int\limits_\omega \varphi_1(\omega)\varphi_2(\omega)\,d\omega \Big/ \int\limits_\omega \varphi_1 d\omega \int\limits_\omega \varphi_2 d\omega \tag{3.2}$$

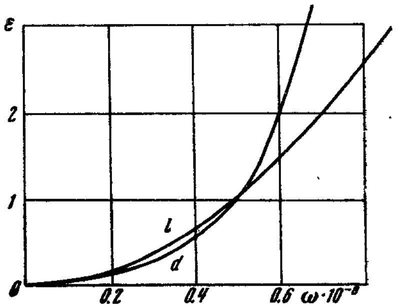

Fig. 2

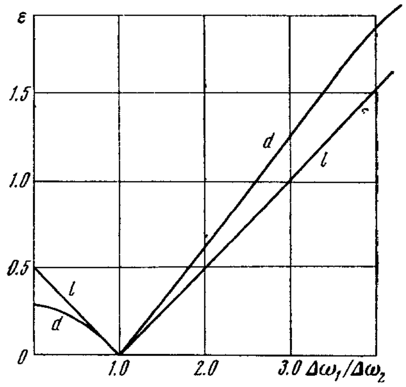

Fig. 3

where the function $\varphi_1(\omega)$ in the numerator defines the relative contribution of amplification at frequency ω to the total amplification, and $\int\varphi_1 d\omega$ is introduced to preserve the normalization introduced above. We will consider several special cases. In the absence of any spectral overlap of $\varphi_1(\omega)$ and $\varphi_2(\omega)$ the integral

$$\int \varphi_1(\omega)\,\varphi_2(\omega)\,d\omega = 0, \quad \varkappa_2 = 0$$

An analogous result occurs in the case where the radiation source possesses a continuous spectrum, i.e., over the limits of a frequency interval significantly exceeding the width of the amplification line $\varphi_1(\omega) =$ const. In this case it follows from Eq. (3.2) that

$$\varkappa_2 = J_{ik} \Big/ \int_\omega d\omega \to 0$$

Of greatest practical interest is the case where the centers of the contours $\varphi_1(\omega)$ and $\varphi_2(\omega)$ coincide. In this case it is possible to use simultaneously two expressions of the type of Eqs. (2.2), (2.3), written with the assumption that $\omega_{01} = \omega_{02} = 0$. The numerical value of Eq. (3.2) then depends on the character of the broadening, as well as on the width itself of the lines $\varphi_1(\omega)$ and $\varphi_2(\omega)$. We first define integrals of the type

$$\int_{-\infty}^{\infty} \varphi(\omega)\,d\omega = 2\int_0^{\infty} \varphi(\omega)\,d\omega$$

for the case of homogeneous and Doppler broadening [1]:

$$\int_0^{\infty} \varphi(\omega)\,d\omega = \begin{cases} \pi\Delta\omega_e/4 \\ \sqrt{\pi}\,\Delta\omega_d/4\sqrt{\ln 2} \end{cases} \tag{3.3}$$

If both lines $\varphi_1(\omega)$ and $\varphi_2(\omega)$ are broadened homogeneously, then from Eqs. (3.2) and (3.3) we have

$$\varkappa_2 = \frac{8J_{ik}I_1}{\pi^2\Delta\omega_1\Delta\omega_2}, \quad I_1 = \int_0^{\infty} \frac{d\omega}{\left[1+4\left(\frac{\omega}{\Delta\omega_1}\right)^2\right]\left[1+4\left(\frac{\omega}{\Delta\omega_2}\right)^2\right]} \tag{3.4}$$

After transformations we obtain

$$I_1 = \frac{\Delta\omega_1^2\Delta\omega_2^2}{4(\Delta\omega_2^2 - \Delta\omega_1^2)}\left[\int_0^{\infty}\frac{d\omega}{\Delta\omega_1^2/4+\omega^2} - \int_0^{\infty}\frac{d\omega}{\Delta\omega_2^2/4+\omega^2}\right] \tag{3.5}$$

Integration then gives

$$I_1 = \frac{\pi}{4}\frac{\Delta\omega_1\Delta\omega_2}{\Delta\omega_1+\Delta\omega_2} \tag{3.6}$$

Combining Eqs. (3.4) and (3.6), we obtain a simple, physically descriptive result

$$\varkappa_2 = \frac{2J_{ik}}{\pi(\Delta\omega_1+\Delta\omega_2)} \tag{3.7}$$

If $\Delta\omega_1 = \Delta\omega_2$, then

$$\varkappa_2^\circ = J_{ik}/\pi\Delta\omega \tag{3.8}$$

If $\Delta\omega_1 \neq \Delta\omega_2$, then the value of $\varkappa_2$ will be determined by the greater of these quantities.

We will now estimate the error introduced by noncoincidence of the contours $\varphi_1(\omega)$ and $\varphi_2(\omega)$. It is obvious that the index of amplification defined by Eq. (3.8) corresponds to the true unsaturated index of amplification of the medium studied, used in the multimode generation regime (amplification over the entire contour). In fact, in this case by definition $\varphi_1(\omega) = \varphi_2(\omega)$ and $\Delta\omega_1 = \Delta\omega_2$. It thus follows that the error may be determined from Eqs. (3.7) and (3.8):

$$\varepsilon = \frac{\varkappa_2^\circ - \varkappa_2}{\varkappa_2} = \left| \frac{1}{2} \left(\frac{\Delta\omega_1}{\Delta\omega_2} - 1 \right) \right| \tag{3.9}$$

If both contours $\varphi_1(\omega)$ and $\varphi_2(\omega)$ have Doppler broadening

$$\varkappa_2 = \frac{8 \ln 2 \, J_{ik} I_3}{\pi \Delta\omega_1 \Delta\omega_2} \tag{3.10}$$

$$I_3 = \int_0^\infty \exp \left[-4 \ln 2 \omega^2 \left(\frac{1}{\Delta\omega_1^2} + \frac{1}{\Delta\omega_2^2} \right) \right] d\omega$$

After integration,

$$I_3 = \frac{\Delta\omega_1 \Delta\omega_2}{4} \sqrt{\frac{\pi}{\ln 2 \, (\Delta\omega_1^2 + \Delta\omega_2^2)}} \tag{3.11}$$

Combining Eqs. (3.10), (3.11), we obtain

$$\varkappa_2 = 2 J_{ik} \sqrt{\frac{\ln 2}{\pi \, (\Delta\omega_1^2 + \Delta\omega_2^2)}} \tag{3.12}$$

If $\Delta\omega_1 = \Delta\omega_2$, then

$$\varkappa_2^\circ = \frac{2 J_{ik}}{\Delta\omega} \sqrt{\frac{\ln 2}{2\pi}} \tag{3.13}$$

If $\Delta\omega_1 \neq \Delta\omega_2$, the value of $\varkappa_2$ will be essentially determined by the larger of these quantities.

The error in this case is defined from Eqs. (3.12), (3.13):

$$\varepsilon = \left| \sqrt{\frac{1}{2} \left[1 + \left(\frac{\Delta\omega_1}{\Delta\omega_2} \right)^2 \right]} - 1 \right| \tag{3.14}$$

In both cases considered $\varepsilon = 0$ if $\Delta\omega_1 = \Delta\omega_2$. The error ε as a function of $\Delta\omega_1/\Delta\omega_2$ for both homogeneous and Doppler broadening is presented in Fig. 3.

If the source spectral line has Doppler broadening, and the medium line is homogeneously broadened, Eq. (3.2) takes on the form

$$\varkappa_2 = \frac{8 \sqrt{\ln 2} \, J_{ik} I_2}{\pi^{3/2} \Delta\omega_1 \Delta\omega_2} \tag{3.15}$$

$$I_2 = \int_0^\infty \exp \left[-4 \ln 2 \left(\frac{\omega}{\Delta\omega_1} \right)^2 \right] \frac{d\omega}{1 + 4 (\omega/\Delta\omega_2)^2}$$

The integral is the well-known Voigt integral [2]. After transformations we can obtain

$$I_2 = \frac{\pi \Delta\omega_2}{4} \exp \left[4 \ln 2 \left(\frac{\Delta\omega_2}{\Delta\omega_1} \right)^2 \right] \left[1 - \Phi \left(\frac{\Delta\omega_2}{\Delta\omega_1} \sqrt{\ln 2} \right) \right] \tag{3.16}$$

where Φ is the probability integral tabulated, for example, in [3]. Combining Eqs. (3.15), (3.16), we obtain

$$\varkappa_2 = \frac{2 J_{ik} \sqrt{\ln 2}}{\sqrt{\pi} \, \Delta\omega_1} \exp \left[4 \ln 2 \left(\frac{\Delta\omega_2}{\Delta\omega_1} \right)^2 \right] \left[1 - \Phi \left(\frac{\Delta\omega_2}{\Delta\omega_1} \sqrt{\ln 2} \right) \right] \tag{3.17}$$

The error in this case is determined from Eqs. (3.13) and (3.15):

$$\varepsilon = \pi \Delta\omega_1 / 4 \sqrt{\ln 2} \, I_2 - 1 \tag{3.18}$$

It is obvious that for the case where the source line is broadened homogeneously with the medium line possessing Doppler broadening, the expression for $\varkappa_2$ will coincide with Eq. (3.17) with interchange of

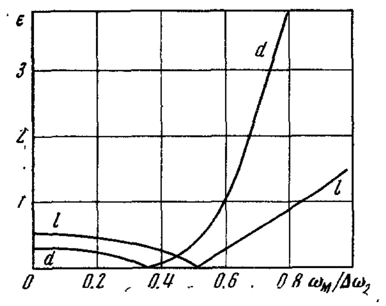

Fig. 4

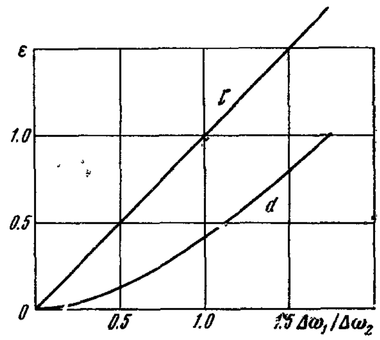

Fig. 5

indices 1 and 2. The error arising in this situation is given by Eqs. (3.8) and (3.15):

$$\varepsilon = \sqrt{\pi}\,\Delta\omega_1/8\sqrt{\ln 2}\,I_2 - 1 \tag{3.19}$$

4. Single-Mode Laser Irradiating Active Medium of Multimode Laser

The index of amplification of the active medium of a multimode laser, as follows from Eq. (3.2), is

$$\varkappa_2^\circ = J_{ik}\int_\omega [\varphi_2(\omega)]^2\,d\omega \Big/ \Big[\int_\omega \varphi_2(\omega)\,d\omega\Big]^2 \tag{4.1}$$

As was shown above, in the particular cases of homogeneous and Doppler broadening this leads to Eqs. (3.8), (3.15). The index of amplification observed upon irradiation differs from these values and is dependent on the displacement of the generation line ω_M relative to the center of the source spectral line. If $\omega_M = 0$, it follows from Eq. (3.1) that

$$\varkappa_2 = J_{ik} \Big/ \int_\omega \varphi_2(\omega)\,d\omega \tag{4.2}$$

If $\omega_M = \pi c/2L$, then

$$\varkappa_2 = J_{ik}\varphi_2(\pi c/2L) \Big/ \int_\omega \varphi_2(\omega)\,d\omega \tag{4.3}$$

From Eqs.(3.1), (3.4) it follows that the relative measurement error

$$\varepsilon = \int_\omega [\varphi_2(\omega)]^2\,d\omega/\varphi_2(\omega_M)\int_\omega \varphi_2(\omega)\,d\omega - 1 \tag{4.4}$$

For a homogeneously broadened contour, from Eqs. (2.2), (3.3), (3.6), and (4.2) we have

$$\varepsilon = |(1 + 4(\omega_M/\Delta\omega_2)^2/2 - 1| \tag{4.5}$$

If $\omega_M = 0$, then $\varepsilon = 1/2$. If $\omega_M \gg \Delta\omega_2$, the measured value $\varkappa_2 \to 0$ and $\varepsilon \to \infty$, while if $\omega_M/\Delta\omega_2 = 1/2$, then $\varepsilon = 0$. For Doppler broadening from Eqs. (2.3), (3.3), (3.1), and (4.2) we have

$$\varepsilon = \frac{1}{\sqrt{2}}\exp\left[4\ln 2\left(\frac{\omega_M}{\Delta\omega_2}\right)^2\right] - 1 \tag{4.6}$$

If $\omega_M \gg \omega_2$, then $\varkappa_2 \to 0$, $\varepsilon \to \infty$ and $\varepsilon = 0$, if $\omega_M/\Delta\omega_2 = 1/2\sqrt{2}$. The error ε as a function of $\Delta\omega_M/\Delta\omega_2$ for the cases of homogeneous and Doppler broadening of the amplification line is presented in Fig. 4. For monochromatic irradiation of a contour broadened simultaneously by two independent mechanisms, described separately by fluctuations $\varphi_1(\omega)$ and $\varphi_2(\omega)$, the total spectrum is given by the packet formula [4]

$$\varphi_2(\omega) = \int_t \varphi_1(t)\,\varphi_2(\omega - t)\,dt \Big/ \int_\omega \varphi_1\,d\omega \tag{4.7}$$

The integral $\int_\omega \varphi_1 d\omega$ is introduced to preserve normalization, while it is obvious in the given case that

$$\int \varphi_1\,d\omega = \int \varphi_2\,d\omega$$

If $\omega = 0$, then

$$\varphi(0) = \int_{\omega} \varphi_1(\omega)\varphi_2(\omega)\,d\omega \Big/ \int_{\omega} \varphi d\omega \tag{4.8}$$

which in combination with Eq. (3.1) gives an expression for determination of $\varkappa_2$ analogous to Eq. (3.2). It then follows that upon irradiation of the amplification contour (multimode regime) by the probe contour (in the case of a laser this implies the multimode regime) the measured index of amplification will appear the same as if a monochromatic source were to irradiate the center of an amplification contour developed from the functions $\varphi_1(\omega)$ and $\varphi_2(\omega)$, describing the form of the source line and the amplification line of the medium examined.

5. Multimode Laser Irradiating Active Medium of Single-Mode Laser

The index of amplification of the medium in the center of the amplification line in this case will be described by Eq. (4.2), and the measured index of amplification will be given by Eq. (3.2). Hence, it follows that the error is

$$\varepsilon = \int_{\omega} \varphi_1(\omega)\,d\omega \Big/ \int_{\omega} \varphi_1(\omega)\varphi_2(\omega)\,d\omega - 1 \tag{5.1}$$

For homogeneously broadened lines $\varphi_1(\omega)$ and $\varphi_2(\omega)$ it follows from Eqs. (3.3), (3.6), and (5.1) that

$$\varepsilon = |\Delta\omega_1/\Delta\omega_2| \tag{5.2}$$

For Doppler broadening of the lines $\varphi_1(\omega)$ and $\varphi_2(\omega)$ it follows from Eqs. (3.3), (3.11), and (5.1) that

$$\varepsilon = |\sqrt{1 + (\Delta\omega_1/\Delta\omega_2)^2} - 1| \tag{5.3}$$

In both cases $\varepsilon \to 0$ only if $\Delta\omega_1 \ll \Delta\omega_2$. The error ε as a function of $\Delta\omega_1/\Delta\omega_2$ for homogeneous and Doppler broadening with the same broadening mechanisms for $\varphi_1(\omega)$ and $\varphi_2(\omega)$ is presented in Fig. 5. For differing broadening mechanisms it follows from Eqs. (3.3), (3.15), and (5.1) that

$$\varepsilon = \pi\Delta\omega_1/4I_2 - 1 \tag{5.4}$$

for homogeneous broadening of the source line, and

$$\varepsilon = \frac{\pi\Delta\omega_1}{4\sqrt{\ln 2}\, I_2} - 1 \tag{5.5}$$

for Doppler broadening of the source line.

6. Method of Calibrated Losses

This method is based upon the introduction into the resonator of elements having sharply defined losses [5]. The critical loss value at which generation ceases is taken as the amplification coefficient. An undoubtable advantage of this method is the elimination of the problem of noncorrespondence of the probe and studied lines. It is necessary, however, to consider that powerful lasers, as a rule, generate in a multimode regime. At the same time termination of generation must obviously occur in a single-mode regime. This means that errors occur analogous to those considered in Sec. 3.

This effect involves not only axial, but also transverse modes, the critical losses for which are strongly dependent on the order of the mode because of diffraction.

We note that in the irradiation method differences in transverse modes are of little significance, since the frequency shift with different transverse indices is much less than c/2L [1]. Noncorrespondence between measured and real coefficients of amplification can occur with any form of selectivity curve in the introduced losses, i.e., introduction of plan-parallel plates produces interferometer effects, leading to mode selection. Another example of selectivity is the introduction of BaF_2 plates into the resonator of a CO_2 laser, which leads to a sharp change in the generated wavelength from 10.6μ to 9.6μ because of the nearness of the barium fluoride absorption boundary. Moreover, this method is applicable only to media with very high amplifications, in which the critical losses introduced greatly exceed the remaining types of loss. In that case the calibrated-loss method will agree satisfactorily with the irradiation method.

7. Measurement of the Index of Amplification in Molecular Lasers

As an example of the above we will analyze the methods employed in measuring the index of amplification in molecular lasers using carbon dioxide. A characteristic peculiarity of CO_2 lasers is the exceptional diversity of their parameters and methods of active-medium excitation. Since the form and width of amplification lines are determined by molecular velocity and collision frequency, gas temperature and density are most significant. As a probe source it is usual to employ CO_2 lasers with axial discharge in sealed tubes, or with axial injection [6-10]. For such devices the gas temperature is usually that of the room (300°K) with composition CO_2, N_2, He, 1, 2, 5, mm Hg, respectively. In this case Doppler broadening is about 55 MHz. Shock broadening may be determined from the values of specific shock half-widths [11] for CO_2-CO_2 collisions (3.3 MHz), CO_2-N_2 collisions (2.7 MHz), and CO_2–He collisions (2.3 MHz). The total broadening for the above mixture is then ~20 MHz. Lasers of this type usually operate in one axial mode, since $\Delta\nu_M = c/2L \approx 100$ MHz, which increases the line width. It is sufficient, however, for the line width to increase several times for this condition to become inoperative. For simplification of the analysis we may assume a Doppler contour in the probe. The additional error introduced by ignoring shock broadening is [11]

$$\varepsilon = 2\sqrt{\frac{\ln 2}{\pi}}\,\frac{\Delta\omega_l}{d\omega_d}\,p \tag{7.1}$$

where p is the gas pressure. We will now consider active media which are at present widely employed.

A. Electrical-Discharge Lasers with Transverse Injection [12-14]. Temperatures little different from room temperature and pressures of the order of 10-100 mm Hg are characteristic of these devices. At the lower limit of this interval we may assume Doppler broadening; at the upper, homogeneous broadening. At the lower limit $\Delta\omega_1/\Delta\omega_2 \approx 1$, at the upper $\Delta\omega_1/\Delta\omega_2 \approx 4$.

B. Gasdynamic and Chemical Lasers. Room temperature and pressures of ~100 mm Hg are characteristic [15-17]. Then $\Delta\omega_2/\Delta\omega_1 \approx 4$.

C. Pulse-Type TEA Lasers [18-20]. Atmospheric pressure and temperatures of 600-800°K are characteristic. The amplification line contour in these lasers is homogeneously broadened, while $\Delta\omega_2/\Delta\omega_1 \approx 20$-100. A unique quality of these lasers is that due to the great broadening, overlap of lines corresponding to differing vibration–rotation transitions develops [20]. Thus, the real width of the amplification contour may exceed $\Delta\omega_2$ by a factor of tens.

The analysis performed in this study has shown that the question of the methodology of amplification-coefficient measurement must not be treated without consideration of the modal composition of generation. Depending on this mode structure increase in $\Delta\omega_2/\Delta\omega_1$ may lead to improved (Sec. 2) or degraded (Sec. 3) accuracy. We have evaluated the errors which may develop due to lack of control over what vibration–rotation transitions are involved in generation.* In a number of cases analysis of the error developing from this cause may be performed in a manner analogous to that used for differing modal composition. For the probe laser this may be accomplished by substitution of axial modes for spectral lines. When relatively low-power lasers are used for probing, it is quite possible that generation due to concurrence of vibration–rotation transitions will occur on only the most powerful line P (20). However, for the medium being studied such an analogy is inapplicable, since the amplification contour is the envelope of separate modes, but not of separate lines. An exception is the TEA laser, in which the individual lines overlap into a continuous band.

In conclusion we will turn to the question of the reliability of studies where the amplification coefficient of CO_2 lasers has been measured without spectral [6-8] or modal control (all studies known to the authors, for example, [6-10]). As was shown above, in many cases the errors are reduced by laser generation on one line in one mode. However, even then there remain errors related to uncertainty in the frequency of the axial mode within the limits c/4L and errors connected with ambiguity in interpretation of the value of index of amplification obtained due to ignoring the generation regime in which it is proposed to utilize the medium studied. Analysis of such errors was performed in Secs. 1 and 3. The index of amplification measured in these experiments defines within an error given by Eqs. (2)-(4) the index of amplification of the medium in the single-mode regime, while the error is smaller, the greater $\Delta\omega_2/\Delta\omega_1$, i.e., in the final consideration, p_2/p_1, where p_2 and p_1 are the gas pressures in the medium studied and the radiation sources.† This fact explains the satisfactory agreement of the results obtained in [6-10]. To convert the experimental data to multimode-generation regimes, Eqs. (4.5), (4.6) (Fig. 4), and Eq. (4.8) should be used.

*It is to be understood that the question of which of the vibrational modes 00°1-10°0 or 00°1-02°0 occurs in generation has been clarified experimentally.

†At pressures above 10 mm Hg, homogeneous broadening predominates.

In general then, it is necessary to give more attention to careful control over spectral and modal composition. The first of these may be accomplished with diffraction devices and the second, with interference devices of high resolving power (for example, Fabry–Perot interferometers). Stabilization of the mode position may be accomplished by adjustment of resonator length (for example, with piezoceramics) [21].

LITERATURE CITED

1. E. F. Ishchenko and Yu. M. Klitkov, Optical Quantum Generators [in Russian], Sovet-skoe Radio, Moscow (1968).
2. S. E. Frish, Optical Atomic Spectra [in Russian], Fizmatgiz, Moscow–Leningrad (1963).
3. E. Jahnke and F. Emde, Tables of Functions with Formulas and Curves, 4th ed., Dover.
4. A. A. Kharkevich, Spectra and Their Analysis [in Russian], Gostekhizdat, Moscow (1953).
5. B. J. Witteman, "Inversion mechanisms, population densities, and coupling-out of a high-power molecular laser," Phillips Res. Repts., 21, No. 2 (1966).
6. G. B. Gembarzhevskii, N. A. Generalov, G. I. Kozlov, and D. I. Roitenburg, "The coefficient of light amplification in a mixture CO_2+N_2+He upon expansion in a supersonic nozzle," Zh. Éksperim. i Tekh. Fiz., 62, No. 3, 844 (1972).
7. G. Lee, F. E. Gowen, and J. R. Hagen, "Gain and power of CO_2 gasdynamic lasers," AIAA J., 10, No. 1 (1972).
8. G. V. Gembarzhevskii, N. A. Generalov, and G. I. Kozlov, "Experimental studies of the coefficient of light amplification in mixtures of $CO_2+N_2+H_2(H_2O)$ upon expansion in a supersonic nozzle," Zh. Prikl. Mekan. i Tekh. Fiz., No. 4,18 (1973).
9. J. F. Figueira, W. H. Reichelt, G. T. Schappert, T. F. Stratton, and C. A. Fenstermacher, "Nanosecond-pulse amplification in electron-beam pumped CO_2 amplifiers," Appl. Phys. Lett., 22, No. 5, 216 (1973).
10. A. M. Robinson, "Spatial resolution of gain in CO_2 transversely excited pulsed discharge," Appl. Phys., 42, No. 10, 4098 (1971).
11. V. V. Danilov, E. P. Kruglyakov, and E. V. Shun'ko, "Measurement of the probability of the transition P 20(00°1–100)° CO_2 and shock broadening upon collisions with CO_2, N_2, and He," Zh. Prikl. Mekhan. Tekh. Fiz., No. 6, (1972).
12. A. E. Hill, "Uniform electrical excitation of large-volume high-pressure near-sonic CO_2-N_2-He flowstreams," Appl. Phys. Lett., 18, No. 5 (1971).
13. C. J. Buczek, R. J. Wayne, P. Chenausky, and R. J. Freiberg, "Magnetically stabilized cross-field CO_2 laser," Appl. Phys. Lett., 16, No. 8 (1970).
14. P. I. Belomestnov, A. I. Ivanchenko, R. I. Soloukhin, and Yu. A. Yakobi, "Use of an extended arc discharge in a CO_2 laser with closed cycle and convective cooling," Zh. Prikl. Mekhan. Tekh. Fiz., No. 1 (1974).
15. E. T. Gerry, Gasdynamic lasers," IEEE Spectrum, 7, No. 11, 51 (1970).
16. V. N. Kroshko and R. I. Soloukhin, "Optimal inversion regimes for excitation of a mixture in supersonic flow," Dokl. Akad. Nauk SSSR, 211, No. 4 (1973).
17. M. S. Dzhidzhoev, V. T. Platonenko, and R. V. Khokhlov, "Chemical lasers," Usp. Fiz. Nauk, 100, No. 4, 641 (1970).
18. J. A. Beaulieu, "High-peak-power gas lasers," Proc. IEEE, 59, No. 4, 667 (1971).
19. Yu. V. Afonin, V. Byshewski, A. G. Ponomarenko, R. I. Soloukhin, and V. N. Tishchenko, "Investigation of the energetic characteristics of powerful laser systems of the electroionization type, in: 11th International Conference on Phenomena in Ionized Gases [in Russian] (1973), p. 166.
20. A. M. Robinson, "Gain distribution in a CO_2 TEA laser," Can. J. Phys., 50, No. 20 (1972).
21. I. M. Beterov, Yu. A. Matyugin, G. A. Milushkin, B. I. Troshin, and V. P. Chebotaev, "A high-stability gas laser," Avtometriya, No. 6, 55 (1972).